

Detection Threshold (DT) for non-Gaussian Backgrounds

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Detection Threshold (DT) in Sonar Equation

- DT is the SNR required to achieve specified P_d & P_{fa}
 - Obtain *detector threshold* (h) from P_{fa}
 - Search for SNR that achieves P_d for given h
- Albersheim [1982] developed algebraic approximations for non-fluct. target (NFT)
 - Included incoherent summation of n samples
 - Accuracy quite good (order 0.1 dB) over wide range of n , P_d & P_{fa}
- Shnidman [2002], Hmam [2003] & Hmam [2005] improved upon Albersheim's results
 - Increased accuracy, various target types

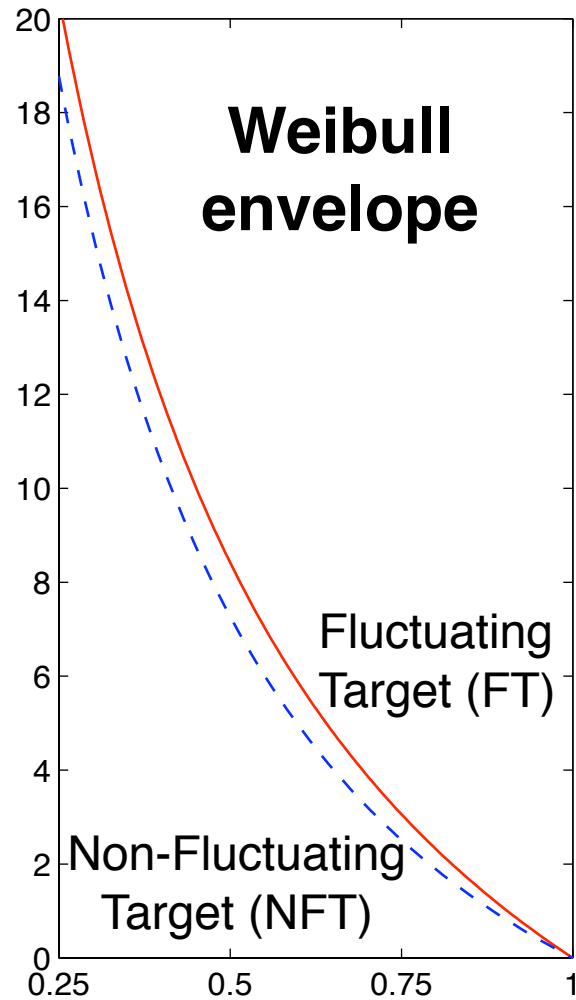
FT in Gaussian
Background
($n = 1$)

$$S = \frac{\log P_{fa}}{\log P_d} - 1$$

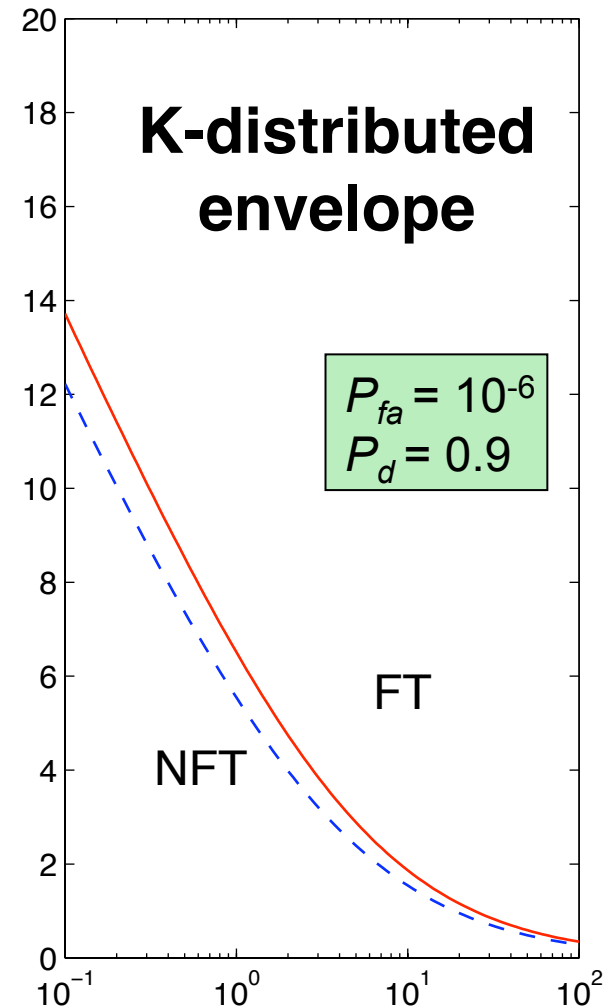
**→ All require Gaussian background
(i.e., Rayleigh envelope)**

How different is non-Gaussian background?

Increase in
DT over DT
for
Gaussian
background
(dB)



Weibull shape param. (β)



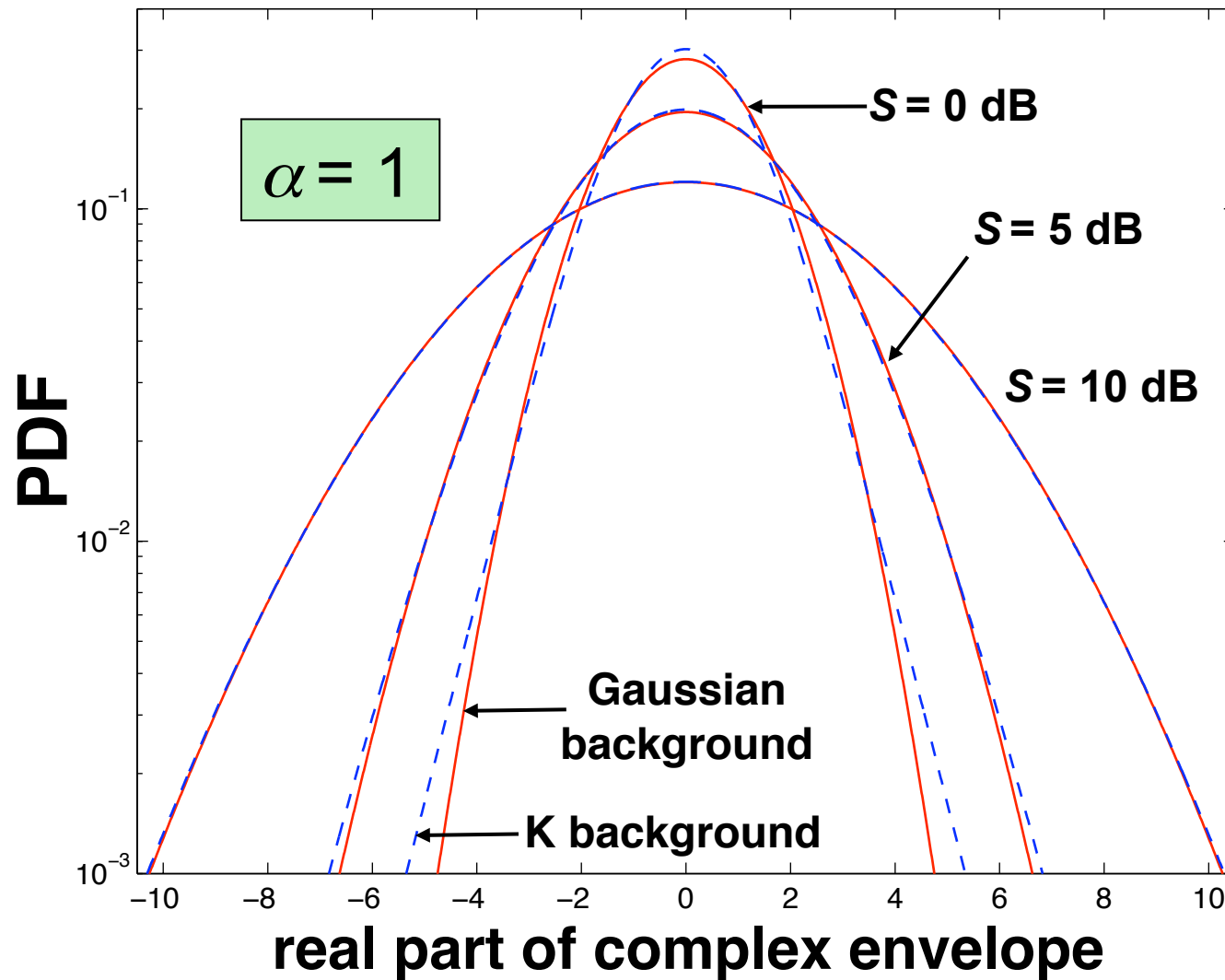
K shape param. (α)

→ Difference can be 5-10 dB in heavy clutter

Detection Threshold in Non-Gaussian Noise

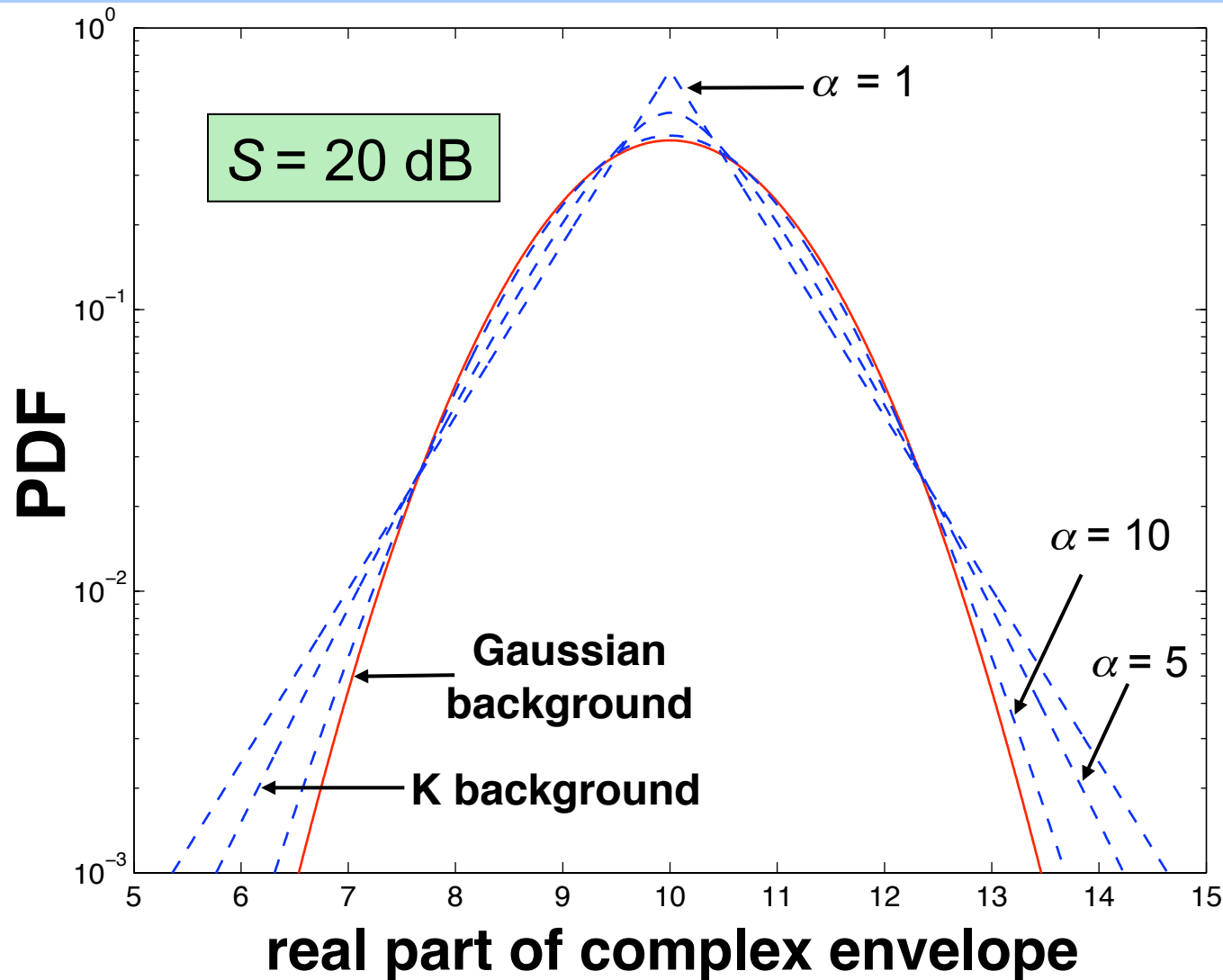
- How to modify for non-Gaussian backgrounds?
 - (1) Use correct relationship between P_{fa} & h
 - Weibull: closed form inversion of CDF
 - K: requires iteration or approximation
 - (2) Approximate P_d via Gaussian background case
 - Compare PDF of real part of complex envelope on next slides
 - For FT, variability in target amplitude dwarfs additional randomness of non-Gaussian background
 - For NFT, target amplitude is constant---background has a bigger impact → use corrections to improve accuracy
 - Only consider $n = 1$ case---should generalize

Fit of Pd approximation: Fluctuating Target



→ Little difference except at very low SNR

Fit of Pd approx: Non-Fluctuating Target



→ Changing SNR only shifts PDF left/right

→ PDFs differ (even high SNR), but structured

How is “ground truth” determined?

- Range of P_d/P_{fa} evaluated:
 - P_{fa} : 10^{-10} to 10^{-2}
 - P_d : 0.1 to 0.98
- Direct evaluation requires a search over a function containing a numerical integral (1- or 2-D, depending on target type) or infinite summation representation
 - Approximations exist; however, they have accuracy issues when background is very heavy tailed
- **Alternative:** simulate data and search over SNR using P_d computed via Monte-Carlo integration (1-D)
 - Using 10^6 samples, standard deviation of error is < 0.01 dB for all cases considered except for
 - Case of $P_d=0.1$ & $P_{fa}=10^{-2}$ for which it was 0.011 dB (both FT & NFT)
 - FT when $P_d>0.8$, rising to 0.031 dB at $P_d=0.98 \rightarrow P_d=0.99$ not evaluated

Weibull Background

- Detector threshold

$$\tilde{h}_W = \frac{(-\log P_{fa})^{1/\beta}}{\Gamma(1 + 1/\beta)}$$

- DT for fluctuating target

$$S \approx \frac{(-\log P_{fa})^{1/\beta}}{-\Gamma(1 + 1/\beta) \log P_d} - 1$$

- DT for non-fluctuating target correction term

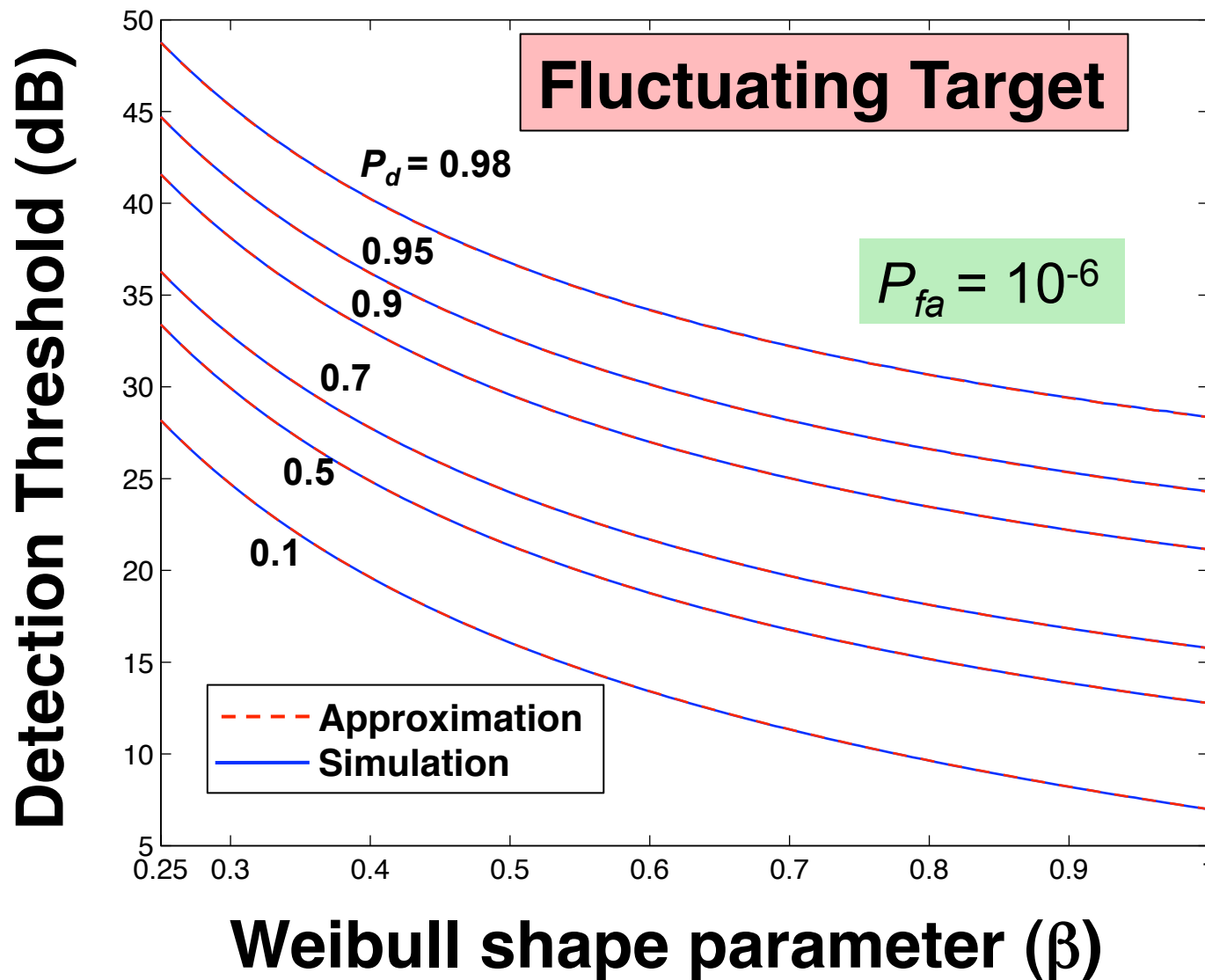
$$S \approx S_H \left(\tilde{h}_W, P'_d(P_d, \beta), 1 \right) \quad P'_d(P_d, \beta) = P_d^{\beta(0.5 - P_d)}$$

- Where Hmam's [2005] approximation is

$$S_H(\tilde{h}_0, P_d, n) \approx \left[\sqrt{\tilde{h}_0 - (n/2 - 0.25)} - \text{sign}(0.5 - P_d)\delta \right]^2 - (n/2 - 0.25)$$

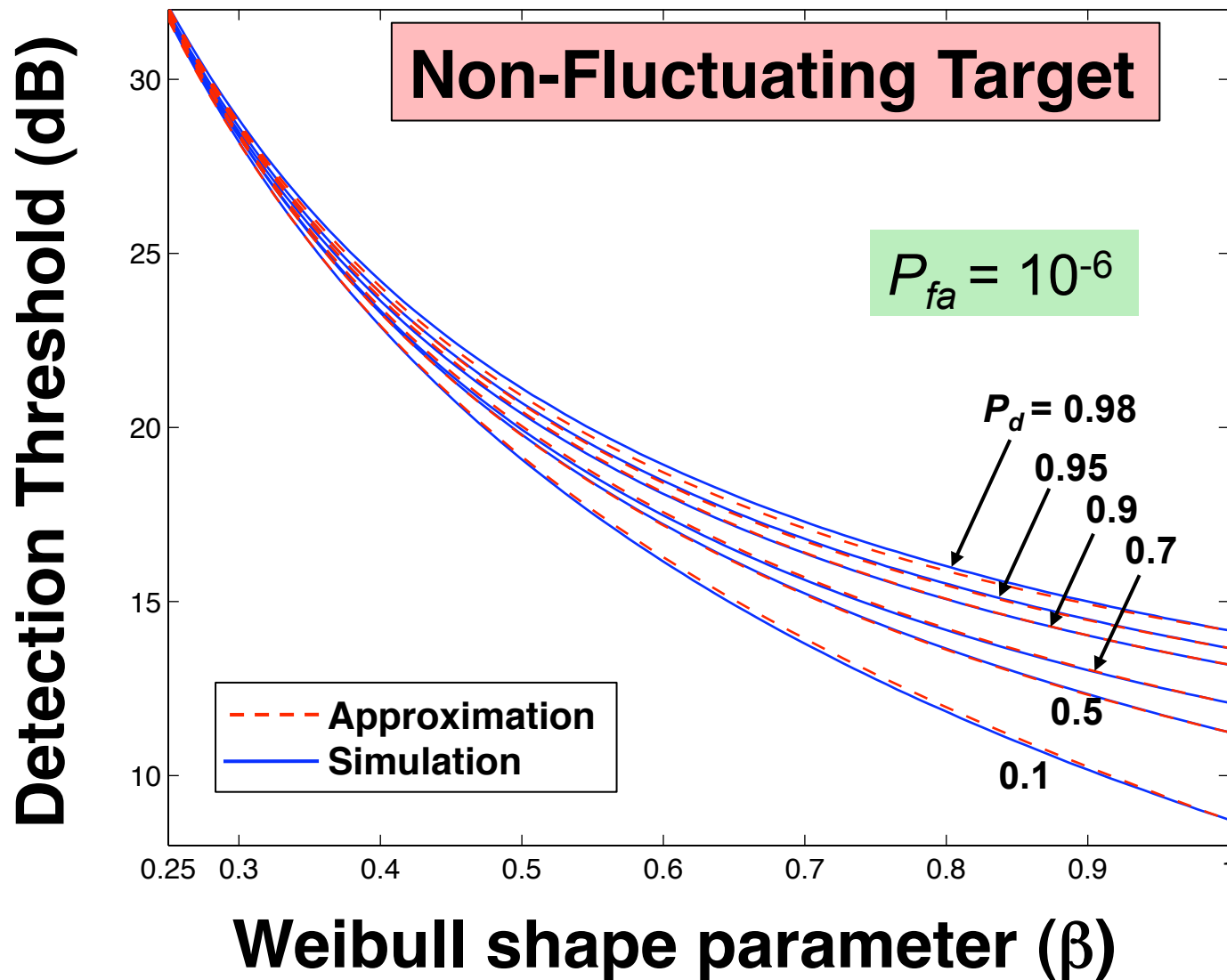
$$\delta = \sqrt{-0.85616 \log \sigma} \quad \sigma = \frac{1}{0.19} \left[\sqrt{0.819025 + 1.5206 P_d(0.9998 - P_d)} - 0.905 \right]$$

Weibull Background: Fluctuating Target



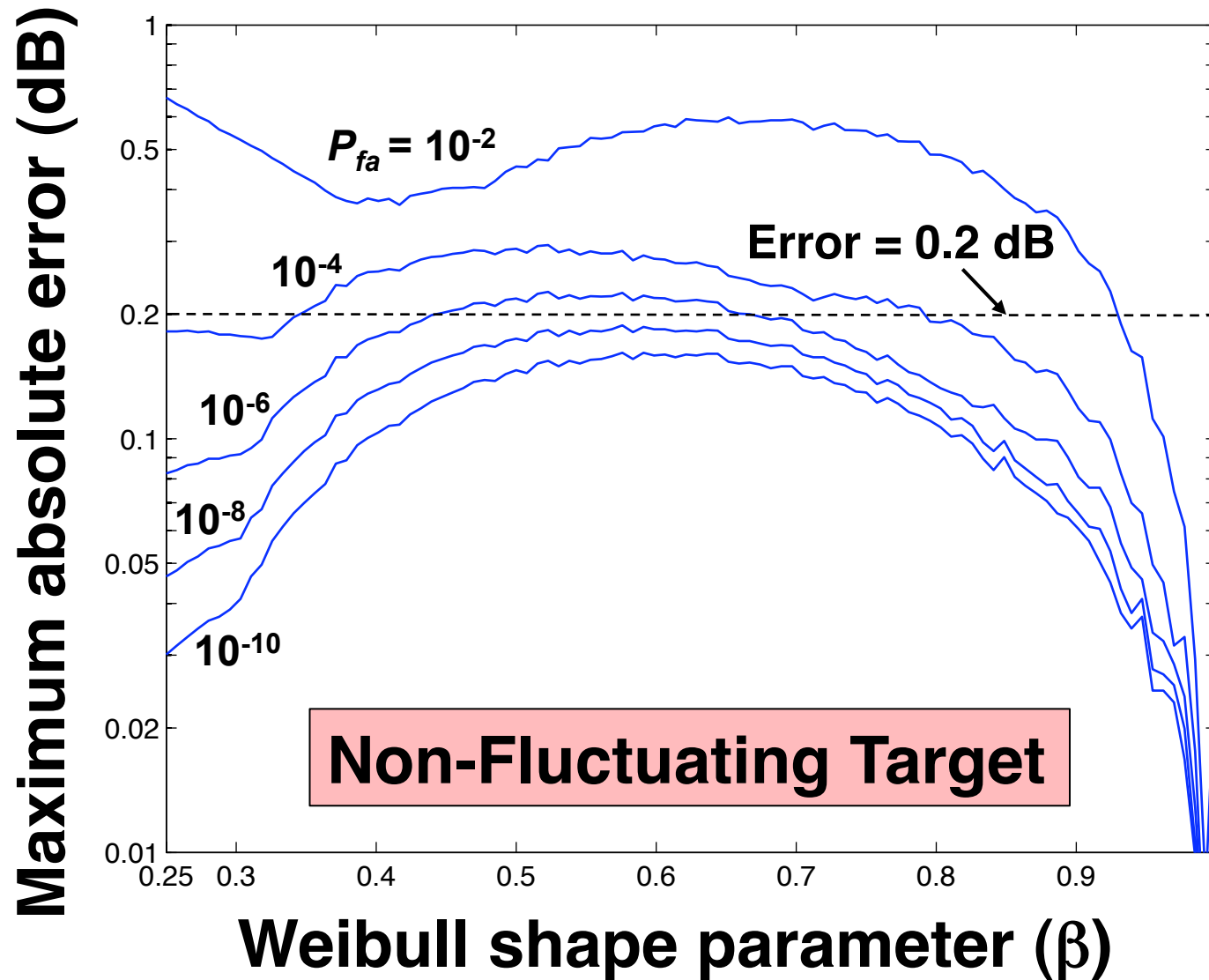
→ Errors < 0.1 dB (~0.04 dB for this P_{fa})

Weibull Background: Non-Fluctuating Target



→ Errors on order of 0.1 dB (~0.2 dB for this P_{fa})

Weibull Background: Non-Fluctuating Target



K-Distributed Background

- Detector threshold

- Exact via Newton-Raphson iteration

- Approximation via

$$\tilde{h}_K \approx \tilde{h}_0 + \frac{c_1 \tilde{h}_0^{c_2}}{\alpha^{c_3} \tilde{h}_0^{0.2}}$$

$$\tilde{h}_0 = h/\sigma_0^2 = -\log P_{fa}$$

P_{fa}	c_1	c_2	c_3	Max error $\alpha > 0.2$
1e-2 to 1e-5	0.099	2.367	0.502	0.179 dB
1e-6 to 1e-10	0.202	2.070	0.479	0.138 dB
1e-6	0.120	2.261	0.500	0.042 dB

- DT for fluctuating target

$$S \approx \left(\frac{\tilde{h}_K}{-\log P_d} - 1 \right) 10^{P_d/200}$$

- DT for non-fluctuating target

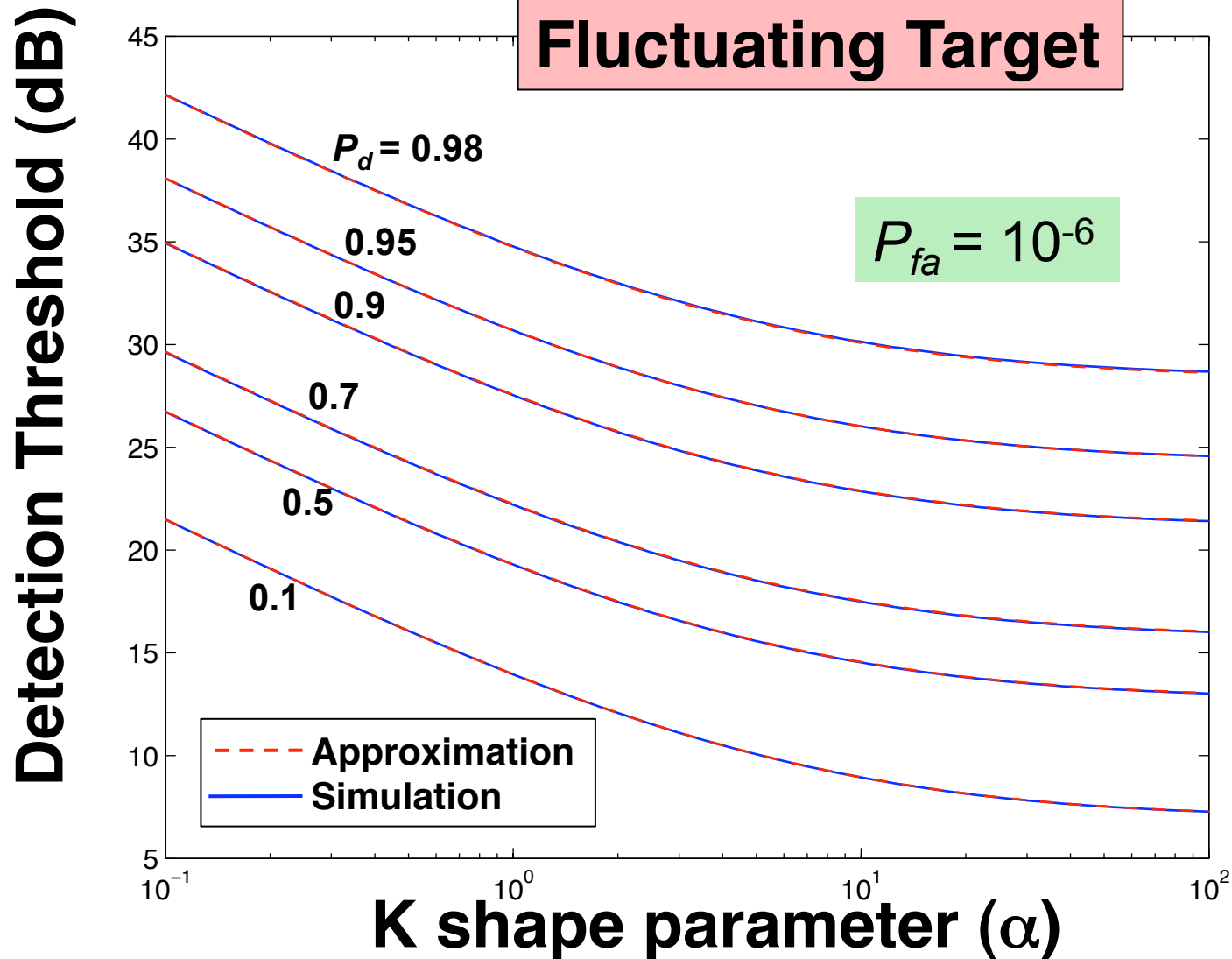
$$S \approx S_H \left(\tilde{h}_K, P'_d(P_d, \beta_K) \right)$$

correction terms

$$\beta_K = 0.3 + 0.7 e^{-\frac{1}{1850} [\log(500/\alpha)]^4}$$

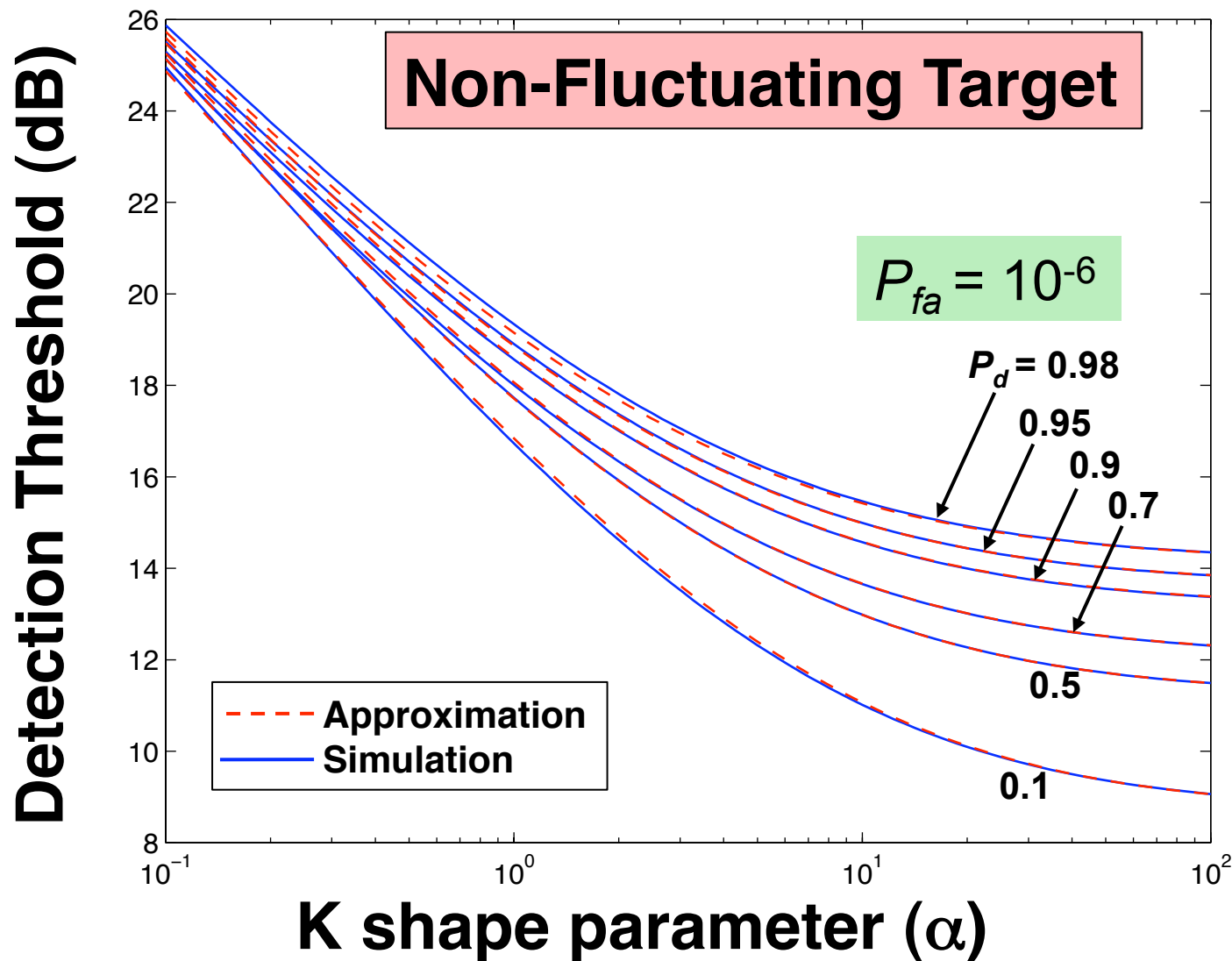
$$P'_d(P_d, \beta) = P_d^{\beta(0.5 - P_d)}$$

K Background: Fluctuating Target



→ Errors < 0.1 dB (~0.06 dB for this P_{fa})

K Background: Non-Fluctuating Target



→ Errors on order of 0.1 dB (~0.2 dB for this P_{fa})

Summary of Approximations

Detector Threshold

Rayleigh Envelope

$$\tilde{h}_0 = -\log P_{fa}$$

Weibull background

$$\tilde{h}_W = \frac{(-\log P_{fa})^{1/\beta}}{\Gamma(1 + 1/\beta)}$$

K background

$$\tilde{h}_K \approx \tilde{h}_0 + \frac{c_1 \tilde{h}_0^{c_2}}{\alpha^{c_3 \tilde{h}_0^{0.2}}}$$

DT for Fluctuating Target

Rayleigh Envelope

$$S = \frac{\log P_{fa}}{\log P_d} - 1$$

Weibull background

$$\approx \frac{(-\log P_{fa})^{1/\beta}}{-\Gamma(1 + 1/\beta) \log P_d} - 1$$

K background

$$\approx \left(\frac{\tilde{h}_K}{-\log P_d} - 1 \right) 10^{P_d/200}$$

DT for Non-Fluctuating Target

Rayleigh Envelope

$$S \approx S_H(\tilde{h}_0, P_d, 1)$$

Weibull background

$$\approx S_H(\tilde{h}_W, P'_d(P_d, \beta), 1)$$

K background

$$\approx S_H(\tilde{h}_K, P'_d(P_d, \beta_K))$$

$$P'_d(P_d, \beta) = P_d^{\beta(0.5 - P_d)}$$

$$\beta_K = 0.3 + 0.7 e^{-\frac{1}{1850} [\log(500/\alpha)]^4}$$

Conclusions

- Extended Hmam's approximation of DT for Gaussian backgrounds to non-Gaussian backgrounds
 - Better than tenth-dB accuracy for fluctuating target (K & Weibull)
 - On order of tenths of dB accuracy for non-fluctuating target
 - High accuracy with minimal effort
- Should extend to $n > 1$ and other background PDFs
 - $n > 1$ ground truth for non-Gauss. background not easily obtained
- Applications
 - Performance prediction in clutter-dominated environments
 - K-threshold approximation useful in implementing adaptive-threshold detector (i.e., statistical normalizer)
 - Approximating PDF of target+clutter by target+Gaussian may be useful in target tracking, classification, or image segmentation
- Journal manuscript (in revision) available at

<http://publications.causasci.com>